PROJECT E O L E

INFORMATIC

STATEMENTS MADE BY THE MATHEMATICS DIVISION

ON MAY 6,1969

Centre National d'Etudes Spatiales
Centre Spatial de Bretigny
Division Mathematiques & Traitement
Departement Calculateurs

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EOLE GENERAL DIFFUSION

P	
DG	
DS	
RE/D - RE/AI - RE/ID	
AF/D - AF/B - AF/OT	
PR/D - PR/PS - PR/BP - O. CAREL	
IG	
CJ	
CT/D	
CT/SDT	
CT/SL	(6 ex.)
CT/BA	(3 ex.)
CT/TT	(2 ex.)
CB/D	
CB/RS	(3 ex.)
CB/ES	(3 ex.)
CB/MT	(5 ex.)

Scientific Director (P. MOREL) (3 ex.)
C.N.R.S. Aeronomy

S U M M A R Y

Introduction

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"EOLE" Project

Informatic

Statements made by the Mathematics Division on May 6, 1969

INTRODUCTION

The informational work accomplished under the responsibility of the Mathematics Division of C.N.E.S., for the EOLE project, is classed in three titles:

1. Studies.
2. Treatment of information for the launching of the satellite.
3. Treatment of the "balloon" and "satellite" information during the operational exploitation phases of EOLE.

In the present account we are interested only in the third point.

The elaboration of the "balloon" and "satellite" operational treatment unfolds in two stages to which two versions of the aforementioned program correspond.

- Version 1 whose objective is to connect the three principal functions of this treatment:

1. The acquisition and the restitution of the two EOLE telemeasures.
2. The calculations of the positions of the balloons and the formation of the data for the scientific treatment accomplished by the dynamic meteorology laboratory of the C.N.R.S.
3. The fabrication of the interrogation program of the balloons (TAF) in direct connection with those responsible for the EOLE operations of the Network division.

This first version should be accomplished for autumn 1969 and should function on data simulated by the Mathematics division and introduced at the level of the same calculator.

- Version 2 whose objective is to properly satisfy the operational needs in actual time:

1. Control of the satellite.
2. Visualization of the positions of the balloons.
3. Regard for the "time" constraint.
4. Regard for the "security of treated and elaborated data" constraint.

This second version should be accomplished for the first global tests which will cause the IRIS telemeasure stations' connections to interfere with Bretigny, the calculator and those responsible for the EOLE operations.

TITLE I

State of progress of the work in Version 1 of the
"balloon" and "satellite" operational treatment.
in the month of May, 1969

Editor: P. Reboul

TITLE I

State of progress of the work of Version 1 of the "balloon" and "satellite" operational treatment in the month of May 1969.

A - The passage of the information.

Figure 1 schematizes the course followed by the information:

A.- 1 The entry of the data.

The satellite transmits its telemeasures to the station which has telecommanded it. These telemeasures are retransmitted to Bretigny by herzienne connection at 200 bauds, in the form of labeled blocks of fixed length. These blocks are acquired by the calculator as soon as they are presented.

A - 2 Treatment of this data.

The two telemeasures are reconstructed separately and memorized. The data which concerns the experimental telemeasure balloons is extracted and the card index of the positions of the balloons is published (BAL).

The data which concerns the fixed nacelles of this same telemeasure is extracted and the card index of the orbit parameters of EOLE is published (SAT).

The dependence telemeasure is operated when one wishes to control the satellite.

A - 3 The results obtained.

The data concerning the positions of balloons and the "nacelle" parameters is reassembled for the scientific treatment.

The knowledge of the satellite orbit and the extrapolation of the balloon positions permits fabrication, under the control of those responsible to Network division, of the teleposting of the satellite for one or several future orbits.

The dependence data furnishes the control of the satellite which the head of the project waits for.

B - Examination of the program which accomplish this work.

B - 1 Principle of programming.

We have relied on two facilities offered by the operation system of the calculator which we shall utilize to accomplish this work.

B - 1.1 The multiprogramming.

Two programs will be executed "together."

The acquisition of the blocks on one hand, which will function 24 h out of 24., and a given one of the treatment programs on the other hand.

B - 1.2 The dynamic structure.

The whole of the treatment programs which will be executed in concomitance with the acquisition will have a dynamic structure.

At any time the resources which this whole will use will be able to be allocated, as a function of their needs, to one or the other of the programs which will succeed.

The order of succession of each program will remain under the control of a calculator operator by means of a connected typewriter (IBM 2740) which we shall call "directing typewriter."

B - 2 State of the work of programming.

Function	Name of the Module	State
Acquisition	ACQ	F, E, T *
Program director of the whole of the treatment programs		F, E, as a test
Restitution of the telemeasures	LIRE	F, E, T
Decommutation and formation of the "nacelle" data	TTLM	
Calculation of the positions of the balloons	PBAL	F, E, T
Calculation of the orbital parameters of the satellite	PSAT	F, in writing
"Balloon-satellite" rendezvous	RVBS	F, E, T
Fabrication of the teleposting	TAF	In formalization
Control of the satellite	CVI	

Integration of the programs above, which exist, will begin in June 1969.

* F = formalize

E = written

T = tested

C - Examination of the material.

C - 1 General organization.

The work which we have described is accomplished in actual time. The two calculators of the Mathematics division of CNES will be utilized.

Figure 2 shows:

C - 1.1 How the telemeasure blocks will be capable of being manually directed, either toward the IBM model 40, or toward the IBM model 65.

C - 2.2 How these two calculators will be able to transmit data from one to the other by means of turntables 2314 and 2311 (two channel switch).

C - 2 Calculator equipment of IBM model 40

The IBM 360 calculator model 40 will assure in priority the EOLE operational treatment.

Figure 3 schematizes its equipment. We have put in dotted lines the material which - we think - will arrive in the course of 1970.

Fig 1

THE PASSAGE OF INFORMATION

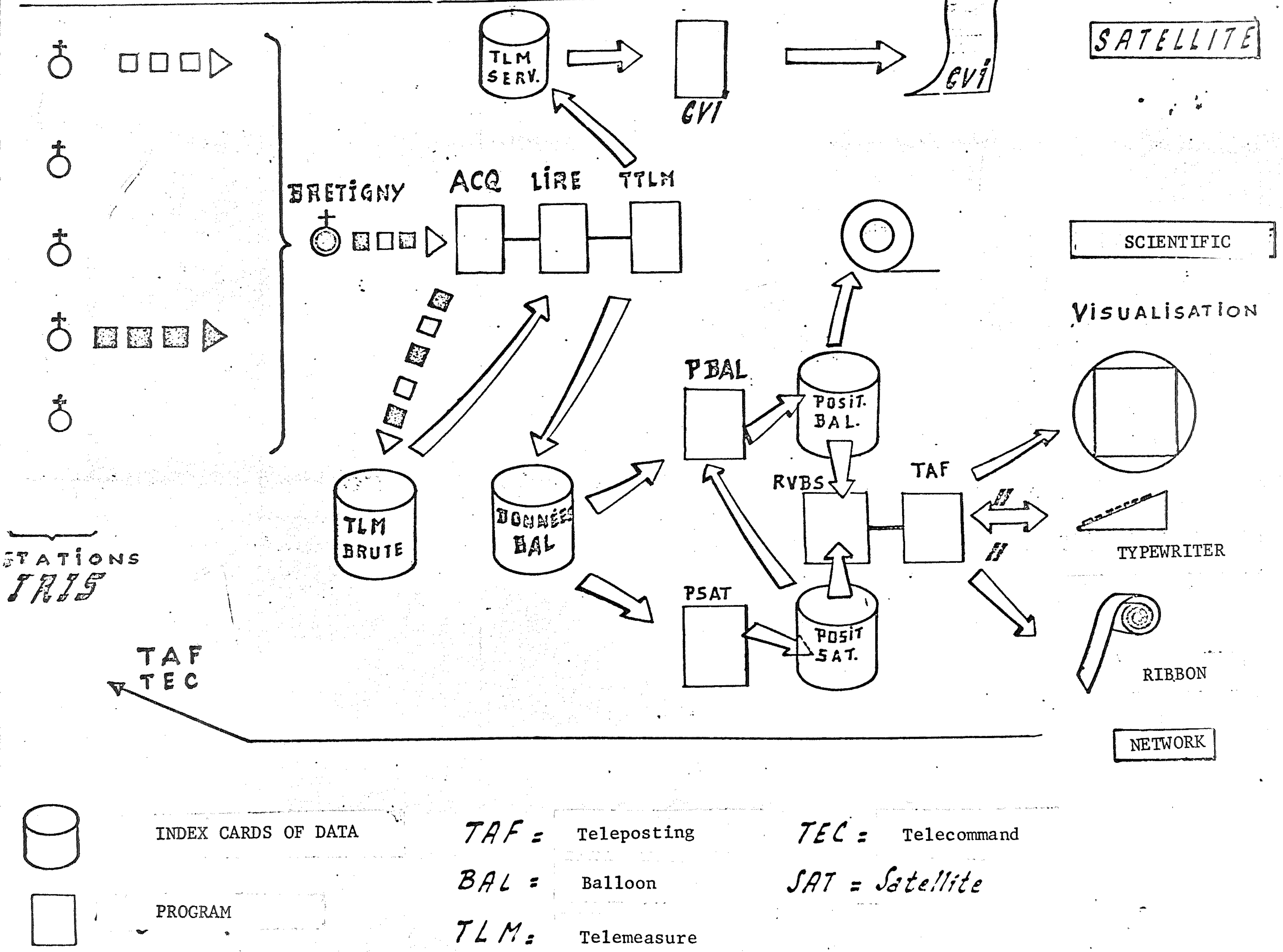
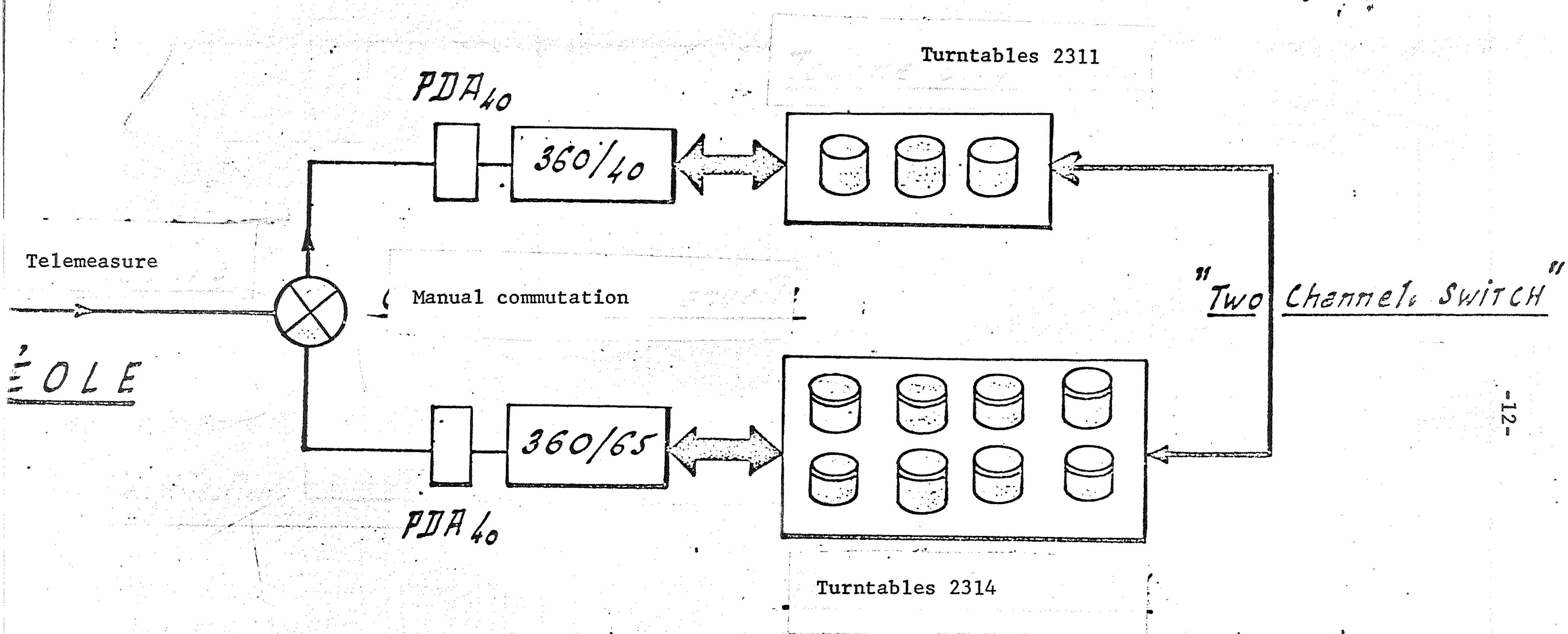


Fig 2

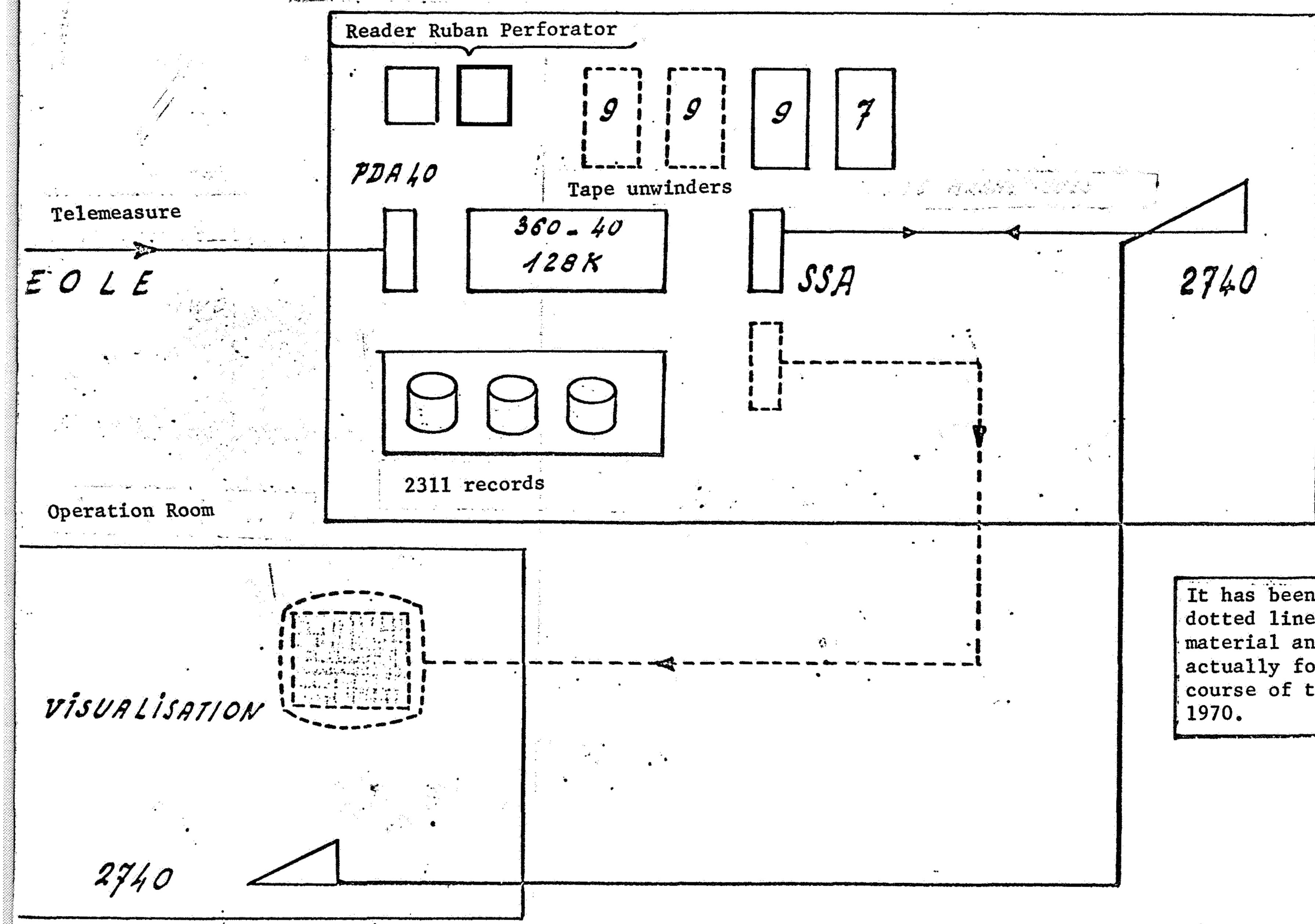
GENERAL VIEW OF THE CALCULATOR EQUIPMENT OF BRETIGNY



PDA_{40} = Adapter of data parallel to 40 wires (IBM)

Fig 3

EQUIPMENT OF THE 360 MODEL 40



It has been noted in dotted lines the material anticipated actually for the course of the year 1970.

SSA : "Start-Stop" adapter

2740 : Distance typewriter

PDA_{40} = Adapter of data parallel to 40 wires

T I T L E II

The recognition of the blocks and the restitution of the telemeasure.

Editor: P. Reboul

TITLE II

The recognition of the blocks and the restitution of the telemeasure.

We are going to study in Title II the programs called ACQ and LIRE in Title I.

Let's remember that we are working under the system MFT II, and that the ACQ program is always in execution in the maximum priority division, while LIRE will be executed in the other division. These two programs are synchronized by means of the table of general conditions which is in the nucleous and which defines the condition of the card indexes and which have been valued the passage of telemeasures received block by block.

Figure 4 schematizes this general organization.

ACQ accomplishes what we call the RECOGNITION of the blocks, LIRE the RESTITUTION of the telemeasure passage. We call "passage" the flow of experimental and dependence telemeasure which is gathered by a station at the time of passage of the satellite.

A - The Format

A - 1 The format of the scientific telemeasure.

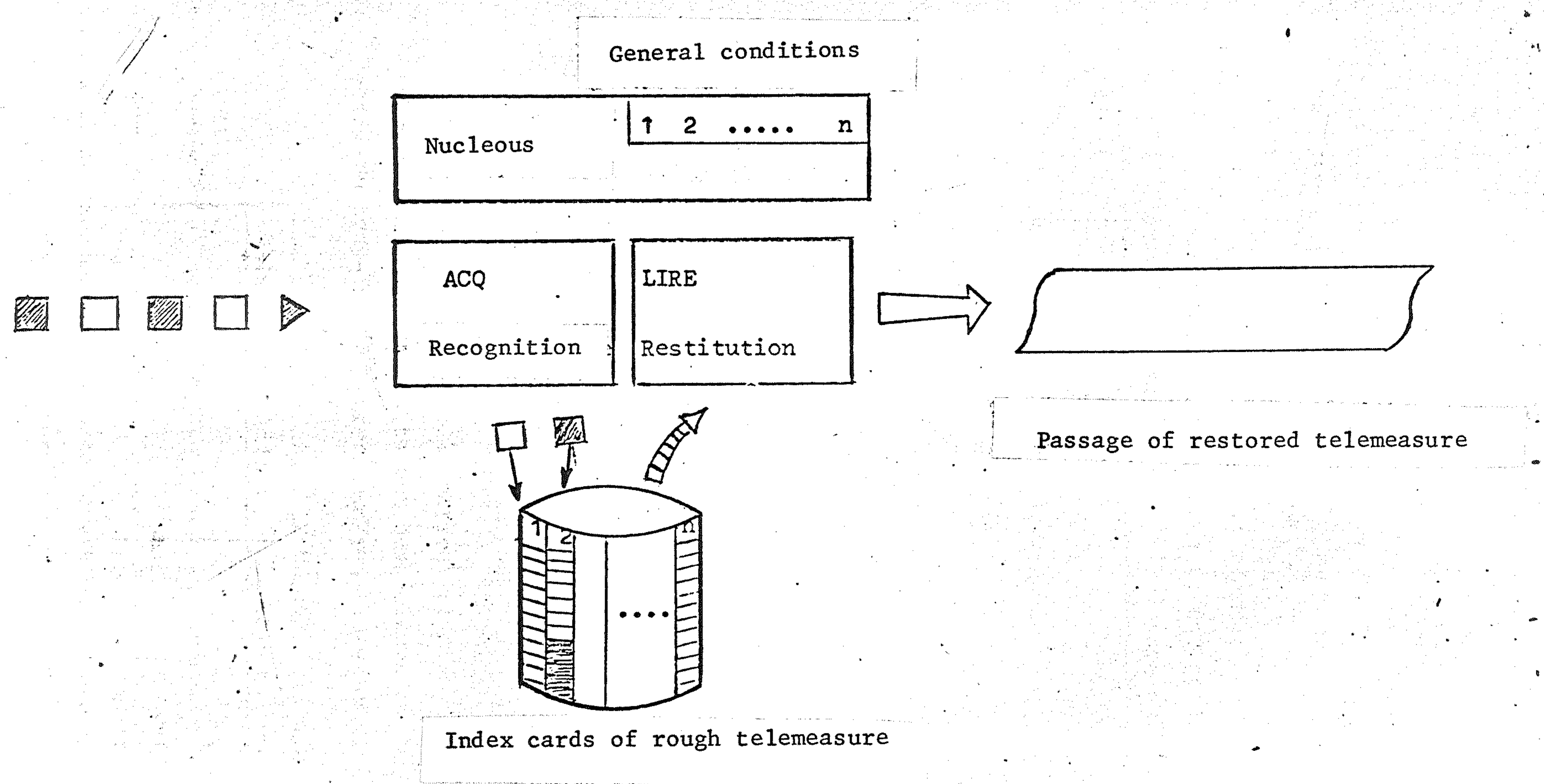
The scientific telemeasure communicates the measures accomplished on board the nacelles and the measures which permit localizing these nacelles. A scientific telemeasure format contains 1056 bits and communicates the corresponding measures to 8 nacelles. (Figure 5).

A - 2 The dependence telemeasure format.

The dependence telemeasure communicates the control data from the satellite. A format of 960 bits in length.

Figure 4

RECOGNITION OF THE BLOCKS AND RESTITUTION OF THE TELEMEASURE



A - 3 The format of the telemeasure block.

A block contains three parts:

- of the telemeasure: 4935 bits which are cut off in the flow of telemeasure at the station;
- a label of 65 bits which is put at the head of the block, at the station;
- a mark of 96 bits (3 words 360) which is put before the label by the ACQ program.

From the quality point of view, the information contained in the mark is not noisy, the information of the label is spoiled with noise from the Bretigny retransmission station and the telemeasure spoiled with noise from retransmission and from the noise of the telemeasure itself.

Figure 6 details the content of the block.

A - 4 The index of rough telemeasure.

The index contains the passages in the course of acquisition for completely acquired, but not yet treated.

This index is composed of N subindexes and an assistance subindex. Each subindex is capable of containing one passage, or the content of two scientific memorandums from the satellite, plus 4 or 5 dependence telemeasure formats. The assistance subindex contains a block switch whose acquisition because of sound can affect the label, have not been able to be recognized and attached to a passage.

The table of general conditions contains two binary positions for each subindex. These two bits have different values according to the four conditions of the corresponding subindex, which are defined in Table 1.

TABLE 1

Value of the 2 bits	Condition of the corresponding subindex
00	Free
01	In the course of filling
10	Assistance subindex
11	Full subindex to be treated

Figure 7 describes the rough telemeasure index. This index is operated by ACQ and by LIRE in direct access by number of registrations.

B The recognition of the blocks

This recognition is accomplished by the ACQ program.

B - 1 Organization

A block is read when it arrives, one recognizes its origin and one stores it simultaneously with the reading of the following block according to the principle of the "flip flop" which we have schematized in organigram 1.

B - 2 The proper recognition.

This recognition is accomplished in accordance with organigram 2. It relies on 3 words of the label.

First (test T1), one is positioned on the label while looking for the two synchros which frame it. If one fails, one declares the block unknown, attaches a label to it and sends it to the assistance subindex. If one succeeds, one tests (T2) the parity of the station number. If this parity is good, the block is furnished with a label and moved to the corresponding subindex. If it is bad, the block is declared unknown and sent to the assistance subindex.

The first test, on data spoiled by noise, has shown that the probability of moving the block of a passage P_1 to a passage P_2 can be considered nil.

A problem remains however posed in the measure where the "closing" of a subindex (that is, the modification of its condition from 01 to 11 and, consequently, the possibility for the treatment division to dispose of it) depends on the recognition of the word of the sign "Last block." If the parity of this word is bad, one will not recognize that the block studied was the last of a passage. Only in a synchronous and external intervention will permit realizing that an open subindex is not yet closed n hours later. One will close it in this case of authority to transmit it to the treatment.

C - <u>Restoration of the telemeasures</u>

This restoration is accomplished by the LIRE program which it executed in the treatment division.

C - 1 <u>Organization</u>.

In order to be able to dispose of a passage, LIRE explores the "general condition" in the research of a subindex marked "1 1". This program is then organized in accordance with organigram 3.

C - 2 <u>The restoration of the telemeasures</u>.

This restoration consists in rereading the subindex treated, block by block, to verify the regularity of the incrementation of the time contained in the label (not spoiled by noise) and of the block numbers (spoiled by noise), to examine the successive blocks of their mark and of their label and to accumulate them. The information thus obtained is the sought after telemeasure. When the incrementation of the block numbers is irregular, one looks again in the assistance sub-index for the missing blocks while trying to frame their time mark between the time marks of the last two blocks of the exploited sub-index.

When the incrementation of the time mark is no longer made in the proper sense, it is that the last block of the passage has been attained.

One expects very good results from this restoration in the case where the noise of retransmission will affect only one label at a time, and not several consecutive labels.

One should expect however to forget a block, notably in the case of the beginning or end of passage block. One considers then taking back by an exterior action the incompleted blocks of the assistance subindex, forgotten in restoration, and to extract the usable interrogations from them.

FIGURE 5

The Format of Scientific Telemeasure

The format = 1056 bits = 8 interrogations.

S	HO3	BO	B1	. . . .	B7

S = format synchro = 24 bits

HO3 = 8 bits of great weight of the word time of 24 bits

BO to B7 = 8 phrases of 128 bits each.

The phrase = 128 bits = 1 interrogation.

Pn	HO1	HO2	Ni	ϕ 1	ϕ 2	ϕ 3	ϕ 4	ϕ 5	ϕ 6	M_1	M_2	M_3	M_4	N	ΔC

Pn	= number of the phrase = 3 bits
HO1, HO2	= 18 bits of slight weight of the word time
Ni	= number of the balloon interrogated
$\phi_1 \ldots \phi_6$	= 6 phase measures for the calculation of the distance
M_1 M_2 M_3 M_4	= parameters sent by the nacelle
N and ΔC	= measures for the calculation of the Doppler effect.

FIGURE 6

The Block

The label = 3 words "360" = 96 bits

T	Fi		E.G.

T = calculator time at the acquisition of the block - 6 octets

Fi = number of subindex where the block is stored - 1 octet

E.G. = general condition at the moment of acquisition of the blocks - 5 octets

The label = 65 bits

Are provided with parity: the block number, the satellite number, the station number, the indicator of the last block.

6	5	4	3	2	1	0	7	6	5

synchro

4	3	2	1	0	P	2	1	0	P

number of the block　　satellite number

11	10	9	8	7	6	5	4	3	2

analogical band number

1	0	2	1	0	2	1	0	P	1

pass number　　station number error

9	8	7	6	5	4	3	2	1	0

hour at the station

						1	P	6	5

date　　last block

4	3	2	1	0

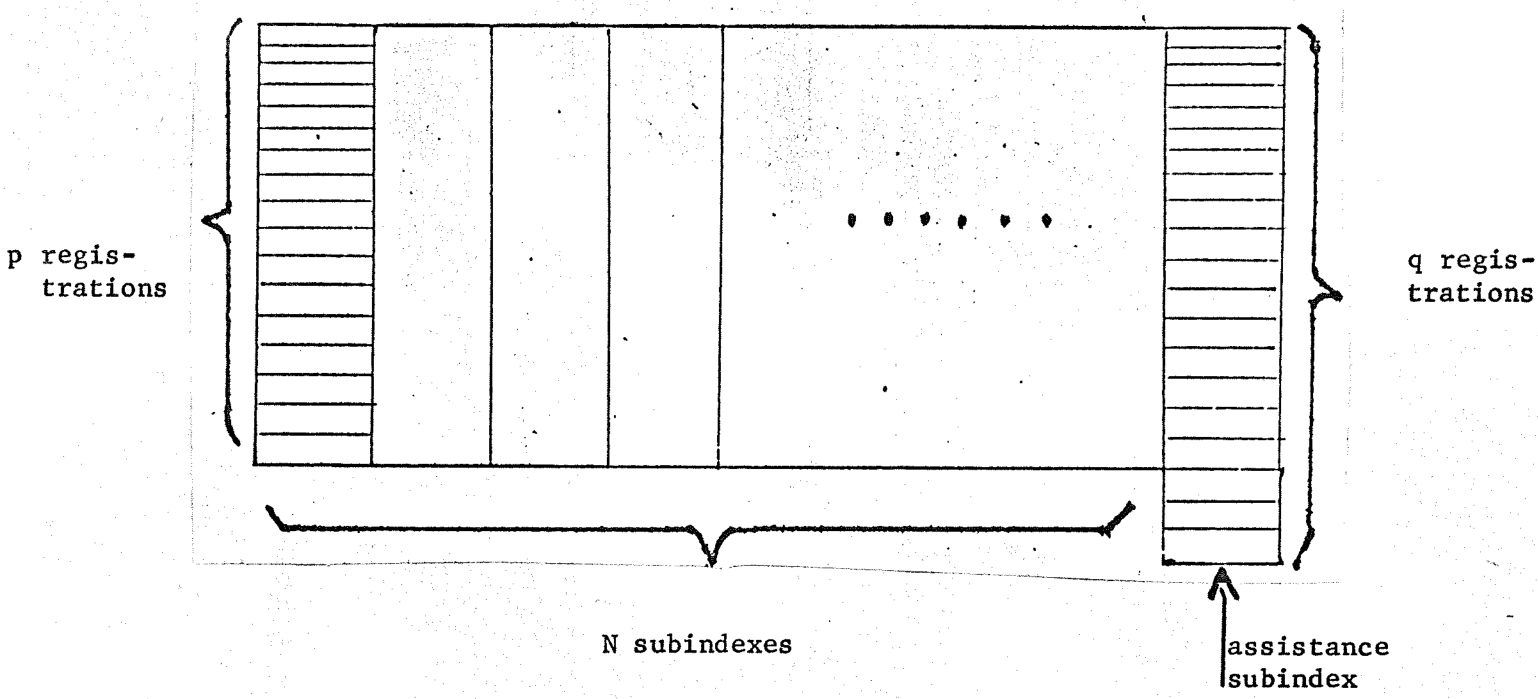
synchro

- The telemeasure: 4,935 bits.
- The complete block makes: 5,096 bits.

FIGURE 7

The Index of Rough Telemeasure

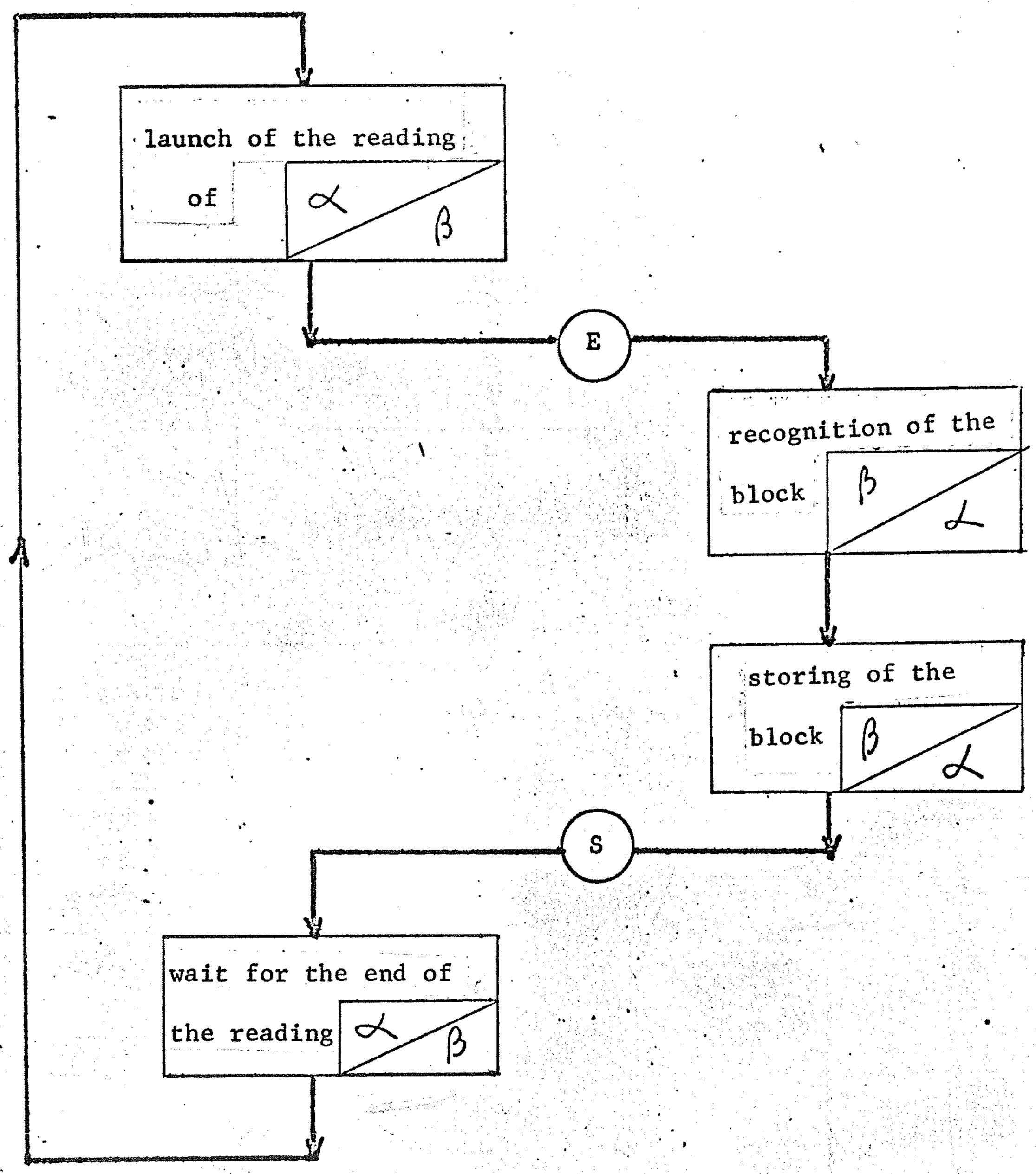
N, p, q, M are fixed quantities which only take the value desired by the programmer at the initialization of the program of rough telemeasure index creation.



Each registration contains M octet, mainly 637 octets in the case of the block described in Figure 6.

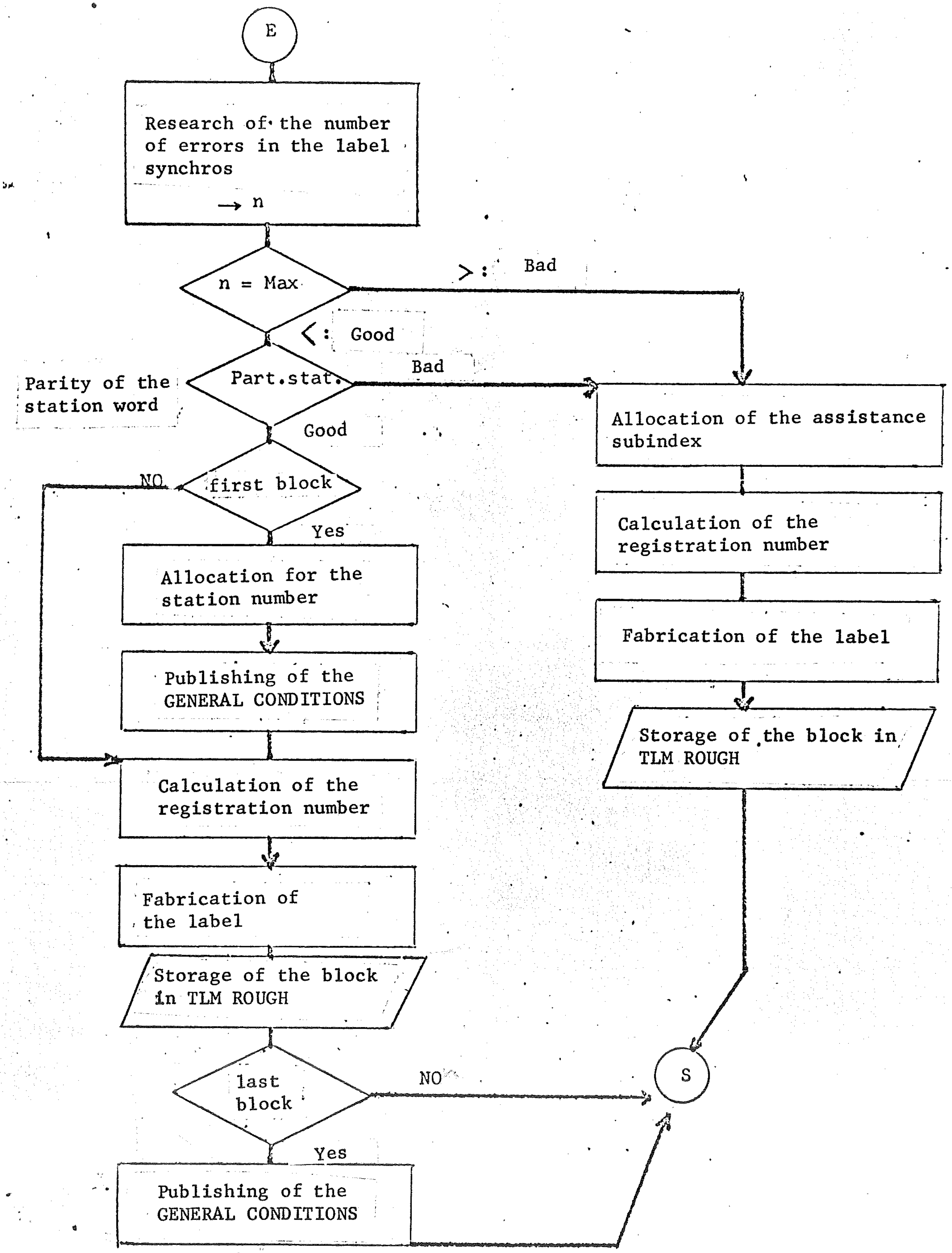
O R G A N I G R A M 1.

General organization of the recognition of the blocks.



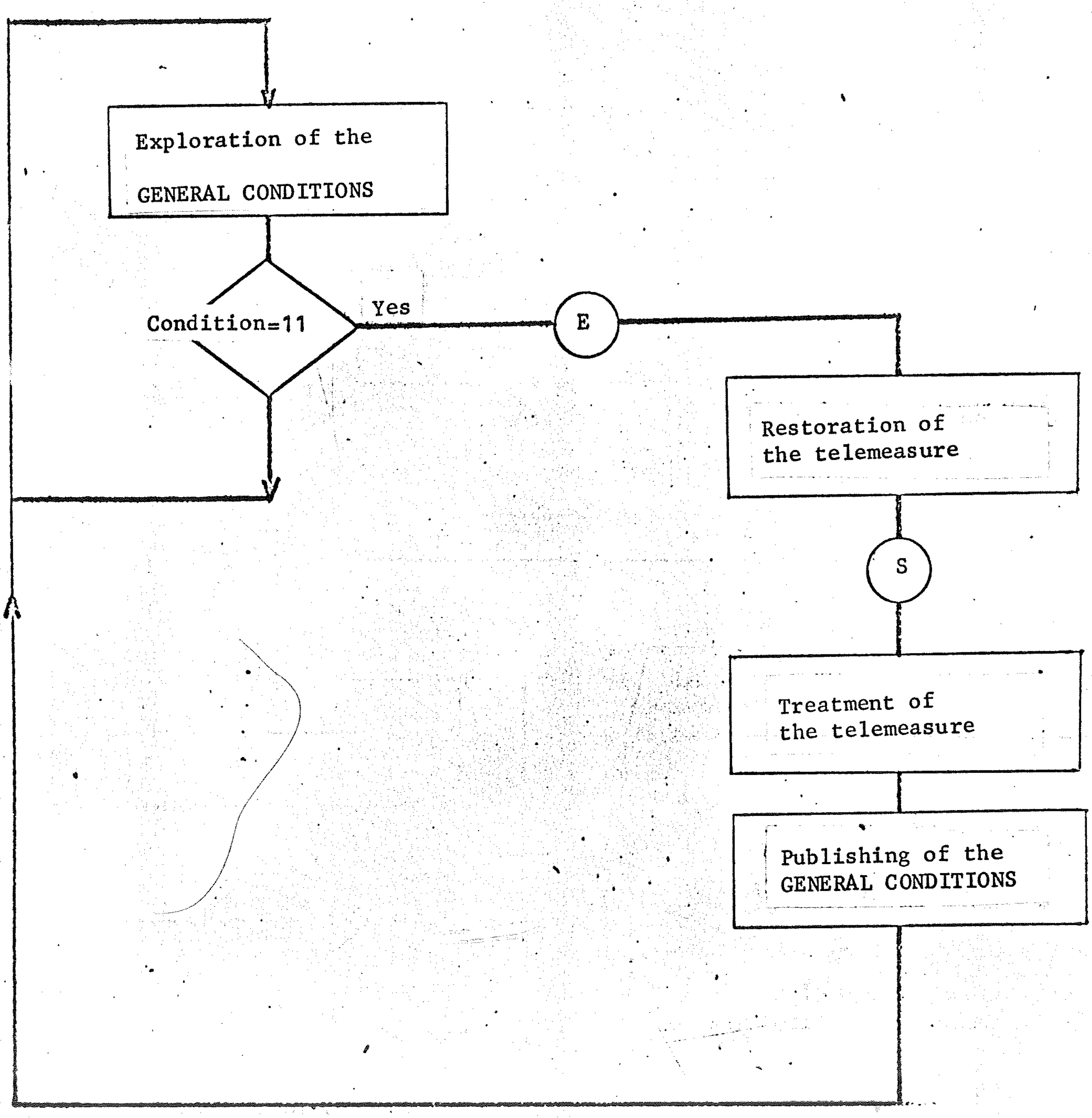
E and S are the input and output of organigram 2.

Recognition of the blocks



ORGANIGRAM 3.

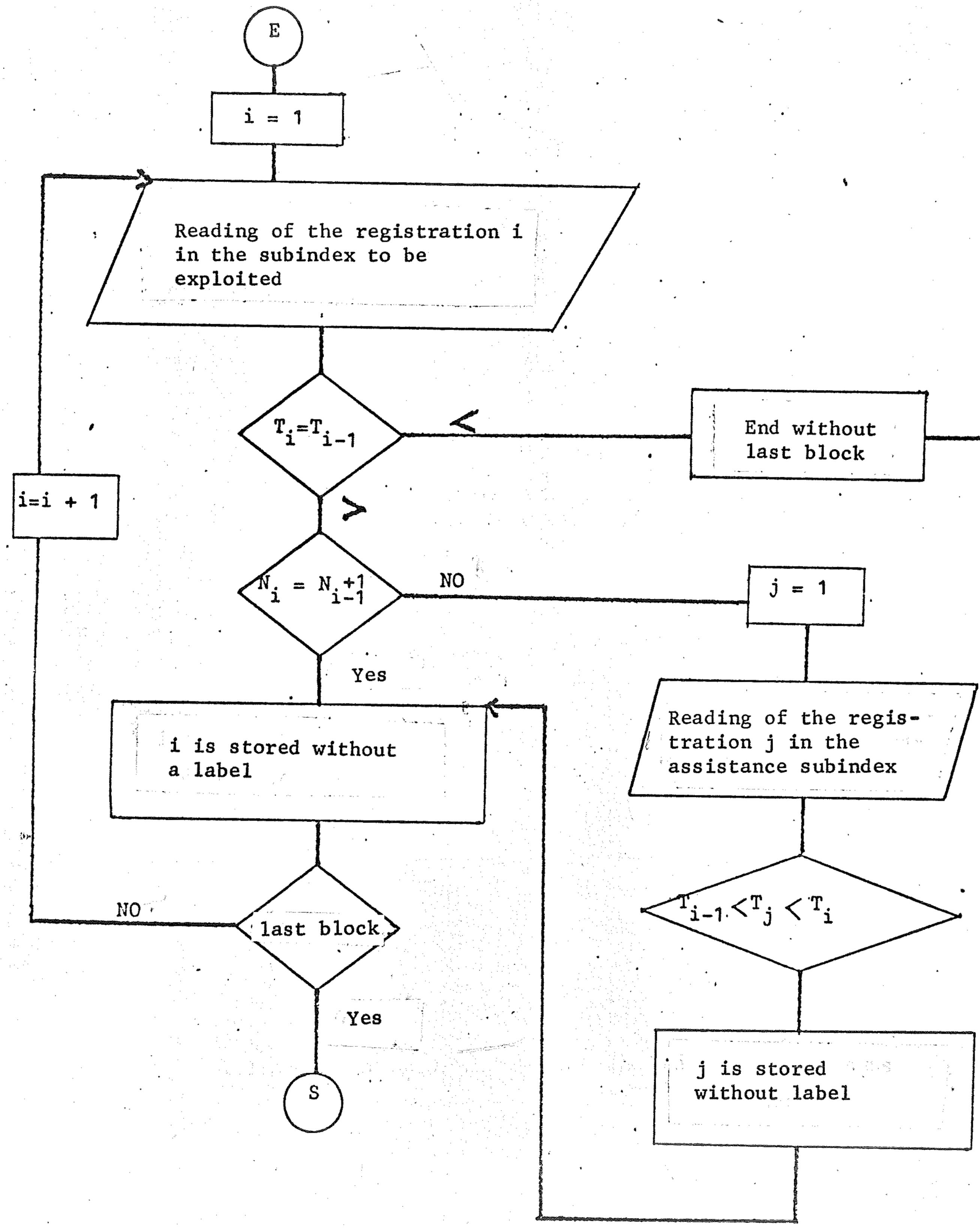
General organization of the restoration of a telemeasure passage



E and S are the input and output of organigram 4.

ORGANIGRAM 4

The restoration of a passage



T_i = Time taken in marking the reception of the block

N_i = Block number taken in the label

T I T L E III

Location Program of the E O L E Balloons

Editress: A. Cazenave

The program calculates the position - and eventually the speed - of each of the balloons in relation to the earth, beginning with Doppler and distance measures, the nominal altitude of the balloon, supposed constant, the position and the speed of the satellite.

The position of the balloon and its speed are defined by five fundamental parameters:

- latitude: φ
- longitude: λ
- altitude: h
- derived in relation to the time of φ : $\dot{\varphi}$
- derived in relation to the time of λ : $\dot{\lambda}$

which can be classed in two categories:

- the free parameters which one hopes to improve in the course of calculation: obligatorily: φ and λ and eventually $\dot{\varphi}$, $\dot{\lambda}$
- the neuter parameters having a fixed value for the series treated.

I - Principle of Location

The method consists of solving a system of approximate equations in operating by linearizations, and repetitions.

Let's designate the quantity measured by M (Doppler or distance) and by F the corresponding theoretical quantity. One obtains an approximate equation in writing that the theoretical quantity is equal to the measured quanity, or:

$$\text{(I)} \qquad F(\varphi, \lambda, h, \dot{\varphi}, \dot{\lambda}, x, y, z, \dot{x}, \dot{y}, \dot{z}) = M$$

where X, Y, Z designate the coordinates of the satellite in a mark chosen for reference, and where $\dot{X}$, $\dot{Y}$, $\dot{Z}$ designate the componants of the satellite vector speed in this same mark.

Two options are possible:

1. The only free parameters are φ and λ ; $\dot{\varphi}$ and $\dot{\lambda}$ are then given by anindex card of field winds.
2. $\varphi, \lambda, \dot{\varphi}, \dot{\lambda}$, are calculated simultaneously.

In the two cases φ and λ are calculated for a given moment t. As for $\dot{\varphi}$ and $\dot{\lambda}$, they are supposed constant in the course of a satellite passage above the horizon of the balloon (in effect one makes the hypothesis that for a maximum duration of 15 minutes the balloon evolves at a constant speed).

Option 1

The parameters to be determined are φ and λ

F not being a linear function of φ and λ , it is necessary to linearize this function in a neighborhood of an approximate solution φ_o, λ_o, the new unknowns being $\delta\varphi_o$, $\delta\lambda_o$.

One obtains the following linearized equation:

$$\frac{\partial F}{\partial \varphi}_{\left(\substack{\varphi=\varphi_o \\ \lambda=\lambda_o}\right)} \delta\varphi_o + \frac{\partial F}{\partial \lambda}_{\left(\substack{\varphi=\varphi_o \\ \lambda=\lambda_o}\right)} \delta\lambda_o = -F(\varphi_o, \lambda_o, h, \dot{\varphi}, \dot{\lambda}, X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}) + M$$

Each satellite-balloon interrogation furnishes two relations of this sort:

- one for the Doppler measure
- one for the distance measure

If the balloon is interrogated n times in the course of a satellite passage above the balloon, one obtains 2n linear equations which one treats by leased squares.

The resolution furnishes $\delta\varphi_0, \delta\lambda_0$ as well as errors to be recognized $\sigma\varphi_0, \sigma\lambda_0$, thus new parameter values:

$$\varphi_1 = \varphi_0 + \delta\varphi_0$$

$$\lambda_1 = \lambda_0 + \delta\lambda_0$$

One begins again the process of repetition by taking these new parameter values as a point of departure, until the convergence is assured, that is, until the values φ_j, λ_j resulting from the same repetition differ - in absolute value - from the true values of a quantity less than ε, ε being a coefficient characterizing the convergence.

Option 2

The parameters to be determined are $\varphi, \lambda, \dot{\varphi}, \dot{\lambda}$ simultaneously. It is not possible then to treat the measures relative to several passages of the satellite or the hypothesis according to which the balloons evolve at a constant speed for a balloon-satellite period of visibility, is no longer acceptable.

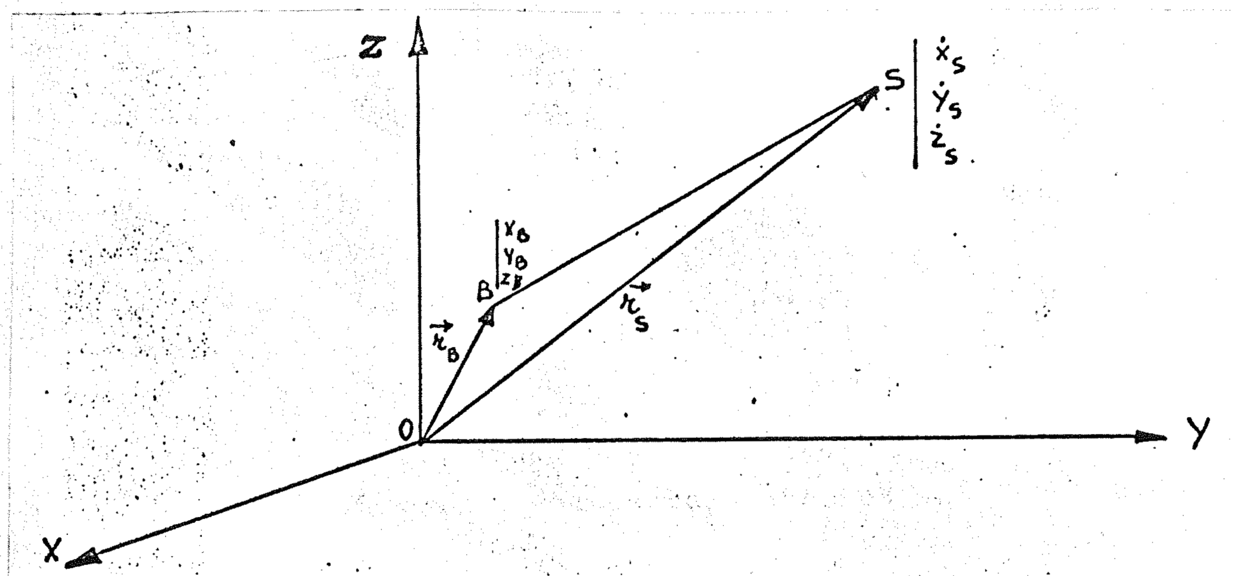
The principle of location is identical to that of Option 1. The linearized equation has the following expression:

$$\frac{\partial F}{\partial \varphi}\,\delta\varphi + \frac{\partial F}{\partial \lambda}\,\delta\lambda + \frac{\partial F}{\partial \dot{\varphi}}\,\delta\dot{\varphi} + \frac{\partial F}{\partial \dot{\lambda}}\,\delta\dot{\lambda} = -F(\varphi_0, \lambda_0, h, x, y, z, \dot{x}, \dot{y}, \dot{z}, \dot{\varphi}_0, \dot{\lambda}_0) + M$$

II. Establishment of the equations of the balloon's position.

One considers a geocentric cartesian mark (T) whose plane XOZ is the plane of the Greenwich meridian.

One calls X_S, Y_S, Z_S, $\dot{X}_S$, $\dot{Y}_S$, $\dot{Z}_S$ the coordinates and the componants of the vector speed in relation to the earth of satellite S.



In like manner one calls:

X_B, Y_B, Z_B, $\dot{X}_B$, $\dot{Y}_B$, $\dot{Z}_B$

the coordinates of a balloon B and the componants of its speed in relation to the earth, in (T).

The Doppler measure furnishes a quantity D such as:

$$\frac{\Delta F}{f} = -\frac{1}{c} D$$

where c is a speed of light and f the reference frequency.

The theoretical expression corresponding to D is:

$$\frac{d}{dt}(\vec{r}_S - \vec{r}_B) = \frac{\vec{SB} \cdot \vec{SB}}{|\vec{SB}|} = \frac{\vec{SB} \cdot (\vec{V}_S - \vec{V}_B)}{|\vec{SB}|}$$

where $\vec{V}_S$ and $\vec{V}_B$ are the vector speeds of S and B, in relation to the earth.

- The distance measure furnishes a quantity d whose theoretical expression is $|\vec{SB}|$

Let's set up $P = |\vec{SB}|$ $\qquad Q = \vec{SB} \cdot (\vec{V}_S - \vec{V}_B)$

P and Q are the theoretical quantities distance and Doppler. The corresponding measured quantities are d and D.

One obtains thus the Doppler and distance equations:

$$
\left\|
\begin{array}{ll}
(1) & \dfrac{Q}{P} = D \\[2em]
(2) & P = d
\end{array}
\right.
$$

- Let's clarify the equations (1) and (2) for a given moment t.

One has:

$$P^2 = (X_S - X_B)^2 + (Y_S - Y_B)^2 + (Z_S - Z_B)^2$$

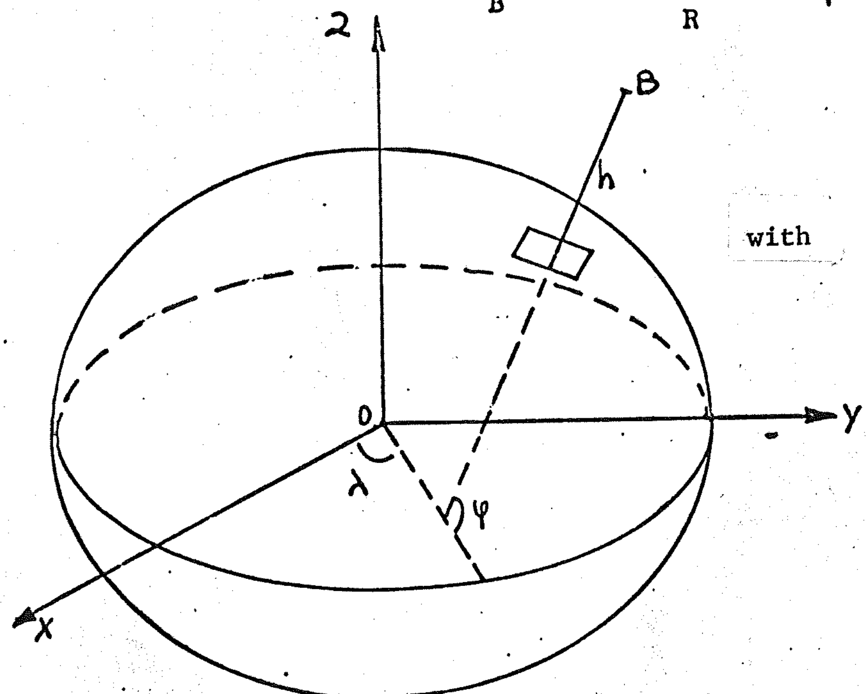
$$Q = (X_S - X_B)(\dot{X}_S - \dot{X}_B) + (Y_S - Y_B)(\dot{Y}_S - \dot{Y}_B) + (Z_S - Z_B)(\dot{Z}_S - \dot{Z}_B)$$

Passage in geographic coordinates φ, λ, h

$$X_B = R(C + \frac{h}{R}) \cos\varphi \cos\lambda$$

$$Y_B = R(C + \frac{h}{R}) \cos\varphi \sin\lambda$$

$$Z_B = R(S + \frac{h}{R}) \sin\varphi$$



R = earth radius

with $c = (1-(2\varepsilon - \varepsilon^2)\sin^2\varphi)^{-1/2}$

$S = (1-\varepsilon)^2 c$

$$\dot{X}_B = R \cos\lambda \left[\cos\varphi \frac{\partial c}{\partial\varphi} - (c + \frac{h}{R})\sin\varphi\right] \dot{\varphi} + Y_B \dot{\lambda}$$

$$\dot{Y}_B = R \sin\lambda \left[\cos\varphi \frac{\partial c}{\partial\varphi} - (c + \frac{h}{R}) \sin\varphi\right] \dot{\varphi} - X_B \dot{\lambda}$$

$$\dot{Z}_B = R \sin\varphi \left[\frac{\partial S}{\partial\varphi} + (S + \frac{h}{R}) \cos\varphi\right] \dot{\varphi}$$

Expression of the linearized equations:

For a treated series, φ and λ are linear functions of the time

$$\varphi = \varphi_A + \dot{\varphi}\,(t - t_A)$$
$$\lambda = \lambda_A + \dot{\lambda}\,(t - t_A)$$

- The linearized equations are written (option 2):

$$\left(\frac{1}{P}\frac{\partial Q}{\partial \varphi} - \frac{Q}{P^2}\frac{\partial P}{\partial \varphi}\right)\delta\varphi + \left(\frac{1}{P}\frac{\partial Q}{\partial \lambda} - \frac{Q}{P^2}\frac{\partial P}{\partial \lambda}\right)\delta\lambda + \left(\frac{1}{P}\frac{\partial Q}{\partial \dot{\lambda}} \; \frac{Q}{P^2}\frac{\partial P}{\partial \dot{\lambda}}\right)\delta\dot{\lambda} + \left(\frac{1}{P}\frac{\partial Q}{\partial \dot{\varphi}} - \frac{Q}{P^2}\frac{\partial P}{\partial \dot{\varphi}}\right.$$
$$= D - \frac{Q}{P}$$

$$\frac{\partial P}{\partial \varphi}\,\delta\varphi + \frac{\partial P}{\partial \lambda}\,\delta\lambda + \frac{\partial P}{\partial \dot{\varphi}}\,\delta\dot{\varphi} + \frac{\partial P}{\partial \dot{\lambda}}\,\delta\dot{\lambda} = d - P$$

These are the condition equations.

- Expression of the partial derivatives:

$$\frac{\partial P}{\partial \varphi} = \frac{1}{P}\left[(X_S - X_B)M(\varphi)\cos\lambda + (Y_S - Y_B)M(\varphi)\sin\varphi + (Z_S - Z_B)N(\varphi)\right]$$

$$\frac{\partial P}{\partial \lambda} = \frac{1}{P}\left[-(X_S - X_B)Y_B + (Y_S - Y_B)X_B\right]$$

$$\frac{\partial Q}{\partial \varphi} = (\dot{X}_S - \dot{X}_B)M(\varphi)\cos\lambda + (Y_S - \dot{Y}_B)M(\varphi)\sin\lambda + (\dot{Z}_S - Z_B)N(\varphi)$$
$$-(X_S - X_B)\frac{\partial \dot{X}_B}{\partial \varphi} - (Y_S - Y_B)\frac{\partial \dot{Y}_B}{\partial \varphi} - (Z_S - Z_B)\frac{\partial \dot{Z}_B}{\partial \varphi}$$

$$\frac{\partial Q}{\partial \lambda} = (\dot{X}_S - \dot{X}_B)Y_B - (\dot{Y}_S - \dot{Y}_B)X_B - (X_S X_B)\frac{\partial \dot{X}_B}{\partial \lambda} - (Y_S - Y_B)\frac{\partial \dot{Y}_B}{\partial \lambda}$$

with:

$$M(\varphi) = R \sin\varphi \left(c + \frac{h}{R}\right) - \cos\varphi \frac{\partial c}{\partial \varphi}$$

$$N(\varphi) = - R \left(S + \frac{h}{R}\right) \cos\varphi - \sin\varphi \frac{\partial S}{\partial \varphi}$$

$$\frac{\partial c}{\partial \varphi} = - \sin\varphi \cos\varphi \,(2\varepsilon - \varepsilon^2)\, c^3$$

$$\frac{\partial S}{\partial \varphi} = (1-\varepsilon)^2 \frac{\partial c}{\partial \varphi}$$

on the other hand:

$$\frac{\partial P}{\partial \dot\varphi} = \frac{\partial P}{\partial \varphi} \frac{\partial \varphi}{\partial \dot\varphi} \quad \text{with} \quad \varphi = \varphi_A + \dot\varphi\,(t-t_A)$$

$$\frac{\partial P}{\partial \dot\lambda} = \frac{\partial P}{\partial \lambda} \frac{\partial \lambda}{\partial \dot\lambda} \quad \text{with} \quad \lambda = \lambda_A + \dot\lambda\,(t-t_A)$$

Finally, Q depending explicitly on $\dot\varphi$ and $\dot\lambda$, one has:

$$\frac{\partial Q}{\partial \dot\varphi} = \frac{\partial Q}{\partial \varphi} \frac{\partial \varphi}{\partial \dot\varphi} + \frac{\partial Q}{\partial \dot\varphi}$$

$$\frac{\partial Q}{\partial \dot\lambda} = \frac{\partial Q}{\partial \lambda} \frac{\partial \lambda}{\partial \dot\lambda} + \frac{\partial Q}{\partial \dot\lambda}$$

finally:

$$\frac{\partial Q}{\partial \dot\varphi} = (t-t_A) \frac{\partial Q}{\partial \varphi} + (X_S-X_B)M(\varphi)\cos\lambda + (Y_S-Y_B)M(\varphi)\sin\lambda + (Z_S-Z_B)N(\varphi)$$

$$\frac{\partial Q}{\partial \dot{\lambda}} = (t-t_A)\frac{\partial Q}{\partial \lambda} + (X_S-X_B)Y_B - (Y_S-Y_B)X_B$$

- Resolution of the linearized system

The matrix A of the partial derivatives is written:

$$A = \begin{bmatrix} \frac{1}{P}\frac{\partial Q}{\partial \varphi} - \frac{Q}{P^2}\frac{\partial P}{\partial \varphi} & \frac{1}{P}\frac{\partial Q}{\partial \lambda} - \frac{Q}{P^2}\frac{\partial P}{\partial \lambda} & \frac{1}{P}\frac{\partial Q}{\partial \dot{\varphi}} - \frac{Q}{P^2}\frac{\partial P}{\partial \dot{\varphi}} & \frac{1}{P}\frac{\partial Q}{\partial \dot{\lambda}} - \frac{Q}{P^2}\frac{\partial P}{\partial \dot{\lambda}} \\ \frac{\partial P}{\partial \varphi} & \frac{\partial P}{\partial \lambda} & \frac{\partial P}{\partial \dot{\varphi}} & \frac{\partial P}{\partial \dot{\lambda}} \end{bmatrix}$$

The matrix B of the second members has as its expression

$$B = \begin{pmatrix} D - \frac{Q}{P} \\ d - P \end{pmatrix}$$

Finally, the matrix X of the unknown parameters is written:

$$\begin{pmatrix} \delta\varphi \\ \delta\lambda \\ \delta\dot{\varphi} \\ \delta\dot{\lambda} \end{pmatrix}$$

- Let P be the matrix of the corresponding weights:

the weight of each condition equation is obtained by linearizing the function F in such a way as to cause the errors on the measured quantities to appear (Doppler, distance, altitude).

$$F(\underbrace{\varphi , \lambda , \dot{\varphi} , \dot{\lambda}}_{\text{quantities sought}} , \underbrace{X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}}_{\text{quantities given}}, \underbrace{h}_{\text{measure}} + \delta h) = \underbrace{M}_{\text{measure}} + \delta M$$

$$\frac{\partial F}{\partial \varphi} \delta\varphi + \frac{\partial F}{\partial \lambda} \delta\lambda \ \frac{\partial F}{\partial \dot{\varphi}} \delta\dot{\varphi} \ \frac{\partial F}{\partial \dot{\lambda}} \delta\dot{\lambda} = - \underset{\text{calculated}}{F} \quad ; + M - \underbrace{\frac{\partial F}{\partial h} \delta h + \delta M}$$

The weight is then equal to the inverse of the sum of the variances of δM and $\frac{\partial F}{\partial h} \delta h$

thus: $$p = \frac{1}{(\frac{\partial F}{\partial h} \delta h)^2 + (\delta M)^2}$$

- let $\tilde{X}$ be an estimation of X, a solution:

$$A^T PA \tilde{X} = A^T PB$$

it comes:

$$\tilde{X} = C^{-1} D$$

with:

$$C^{-1} = [A^T PA]^{-1} , \quad D = A^T PB$$

from whence these solution φ, λ at a moment t of the passage, with the $\sigma\varphi$, $\sigma\lambda$ associated.

And in like manner: $\dot{\varphi}$, $\dot{\lambda}$, $\sigma_{\dot{\varphi}}$, $\sigma_{\dot{\lambda}}$

Remarks

- The initial values (for the repetition) of the free parameters can be obtained in two ways:
  - by an extrapolation program of the balloon positions
  - by the anticipation program
- The positions and the speed of the satellite at the moment of measure are calculated by extrapolation of the last calculated orbital parameters.
- Each result relative to the localization of a balloon consists of the following different parameters:

balloon no.	nb nacelles	wind indicator	date	hour
	having carried this number	0: if $\dot{\varphi}$, $\dot{\lambda}$ are given 1: if they are calculated	day julien	fraction day julien

φ	σ_φ	λ	σ_λ	$\dot{\varphi}$	$\dot{\lambda}$	a	b	c	d
radians	rd	rd	rd	rd/s	rd/s	meteorological parameters			

A simulation has been accomplished.

- The measures have been fabricated with the help of a numerical integration program furnishing at given moments the Doppler and distance measures, between the satellite a balloon (when the latter is visible) as well as the position and speed of the satellite.

- This data has been introduced in the localization program of the balloons, after having affected the measures obtained from hazardous errors, on the order of 250 m for the distances and 0.5 m/s for the Doppler.

Some examples of the cases treated:

first table — the results concern immobile balloons.

second table ---> $\dot{\varphi}$ and $\dot{\lambda}$ are given by the wind index.

The results concern balloons having a uniform circular movement around the earth, their speed being on the order of 100 km/h

third table ---> $\dot{\varphi}$ and $\dot{\lambda}$ are calculated at the same time as φ and λ

BALLOONS HAVING A UNIFORM CIRCULAR MOVEMENT

Number of Interrogations	φ_{A0} λ_{A0}	$\dot\varphi_0$ $\dot\lambda_0$	$\varphi_{A\ real}$ $\lambda_{A\ real}$	$\dot\varphi_{real}$ $\dot\lambda_{real}$	$\varphi_{A\ calculated}$ $\lambda_{A\ calculated}$	$\dot\varphi_{calculated}$ $\dot\lambda_{calculated}$	$\sigma_{\varphi A}$ $\sigma_{\lambda A}$
8	$-47°$ $+135°$	0 0	$-49°$ $+141°$	3.10^{-4} d°/s 3.10^{-4} d°/s	$-49°,0186$ $141°,0186$	$3,2.10^{-4}$ $2,4.10^{-4}$	16.10^{-3} 15.10^{-3}
5	$-48°$ $+140°$	0 0	$-49°$ $+141°$	3.10^{-4} d°/s 3.10^{-4} d°/s	$-48°,979$ $140°,9821$	$2,8.10^{-4}$ $3,5.10^{-4}$	3.10^{-2} 3.10^{-2}
2	$-51°$ $142°$	0 0	$-49°$ $+141°$	3.10^{-4} d°/s 3.10^{-4} d°/s	$-48°,9701$ $140°,969$	$2,9.10^{-4}$ $3,7.10^{-4}$	– –
3	– –	– –	$-1°,81$ $-11°,19$	3.10^{-4} d°/s 3.10^{-4} d°/s	$-1°,809$ $-11°,197$	$3,1.10^{-4}$ $2,9.10^{-4}$	10^{-3} 4.10^{-2}
3	– –	– –	$-1°,70$ $14°,70$	3.10^{-4} d°/s 3.10^{-4} d°/s	$-1°,701$ $14°,693$	$3,1.10^{-4}$ $2,9.10^{-4}$	2.10^{-3} 9.10^{-2}
3	– –	– –	$-6°,81$ $-9°,19$	3.10^{-4} d°/s 3.10^{-4} d°/s	$-6°,810$ $-9°,169$	$2,7.10^{-4}$ $3,3.10^{-4}$	15.10^{-4} 14.10^{-4}
2	– –	– –	$-34°,539$ $9°,54$	3.10^{-4} d°/s 3.10^{-4} d°/s	$-34°,368$ $9°,360$	$6,2.10^{-4}$ $3,2.10^{-4}$	38.10^{-2} 40.10^{-2}
4	– –	– –	$-0°,19$ $12°,81$	3.10^{-4} d°/s 3.10^{-4} d°/s	$0°,21$ $12°,83$	$2,6.10^{-4}$ $3,5.10^{-4}$	7.10^{-2} 8.10^{-2}
3	– –	– –	$-6°,72$ $21°,72$	3.10^{-4} d°/s 3.10^{-4} d°/s	$-6°,700$ $21°,734$	$2,5.10^{-4}$ $3,5.10^{-4}$	14.10^{-2} 11.10^{-2}

BALLOONS HAVING A UNIFORM CIRCULAR MOVEMENT

$\dot{\varphi} = 3.10^{-4}$ d°/s

$\dot{\lambda} = 3.10^{-4}$ d°/s

Number of interrogations	φ real λ real	φ calculated λ calculated	σ_φ σ_λ
4	- 15° - 1°	- 14°,995 - 0°,995	$51.\ 10^{-4}$ degrees 60.10^{-4}
3	5°,25 9°,75	5°,249 9°,748	6.10^{-4} 6.10^{-4}
2	12°,25 0°,75	12°,251 0°,750	17.10^{-4} 22.10^{-4}
1	- 13°,93 - 32°,07	- 13°,933 - 32°,072	
2	14°,142 - 27°,138	14°,143 - 27°,136	25.10^{-4} 22.10^{-4}
3	7°,05 - 23°,05	7°,048 - 23°,050	10^{-3} $1,2.10^{-3}$
3	- 20°,95 - 4°,05	- 20°,951 - 4°,049	7.10^{-4} 8.10^{-4}
4	14°,14 - 1°,14	14°,139 - 1°,139	$2,4.10^{-4}$ $2,8.10^{-4}$

BALLOONS HAVING NO SPEED

Number of interrogations	φ_o λ_o	φ real λ real	φ calculated λ calculated	σ_φ σ_λ
1	-	- 20° - 40°	- 19°,9995 - 39°,9991	10^{-5} degree 2.10^{-5} "
3	-	- 12° 25°	- 11°,998 24°,998	5.10^{-4} 5.10^{-4}
3		- 15° 30°	- 15°,0003 30°,000	19.10^{-4} 25.10^{-4}
2		- 10° - 3°	- 9°,9999 - 3°,0016	7.10^{-4} 6.10^{-4}
5	13° 16°	12°,1280 15°,0160	12°,12606 15°,0200	$1,7.10^{-3}$ 6.10^{-3}
3	- 43° 137°	- 49° 141°	- 48°,99846 141°,00063	6.10^{-3} 4.10^{-3}
8	- 43° 137°	-49° 141°	- 49°,00011 141°,00073	9.10^{-4} 12.10^{-4}
1 subsatellite balloon	- 48° 140°	- 55° 136°	- 54°,99123 135°,99039	
8 error on the altitude:250 m	- 43° 137°	- 49° 141°	- 48°,99723 141°,00047	9.10^{-4} 11.10^{-4}

T I T L E IV

The satellite-balloon rendezvous in

the chain of E O L E treatment.

Editor: S. Golinsky

I. Role in place of the previsions in the chain of EOLE treatment.

I - 1 The EOLE experiment has as its goal collection by satellite of measures permitting reconstruction of the variable positions of a flotilla of balloons. The collection of the information can be done in two ways:

1- by systematic interrogation of the balloons

2- by programmed interrogation of the balloons

The first method of interrogation necessitates only the knowledge of the moment of entry and departure of the satellite from the geographic zone whose points are in a latitude less than 10^{o}. It is not possible however to limit oneself to these simple calculations for:

- it will always be necessary, for scientific or security reasons, to program the interrogation of certain balloons in order to collect the maximum of measures on them;
- it is necessary, in order to supervise the proper continuation of the experiment, to compare the list of the balloons theoretically visible by the satellite with the list of those actually seen.

That which precedes leads to the realization of a preview program giving the balloons-satellite rendezvous, which will take place in the chain of EOLE treatment, between the program restoring the position of the balloons and the program of teleposting. Whatever may be the means of interrogation, it will always be necessary to know what the fixed stations are which can best transmit the orders to the satellites and collect the telemeasure. The preview program should include the calculation of the satellite's passages above fixed stations.

I - 2 It is standard to make passage previews of a satellite above fixed stations. The balloons cannot be considered as fixed stations, they ought to be considered as mobile objects whose position ought to be extrapolated in order to obtain an exact rendezvous with the satellite.

One can consider this extrapolation in two ways:

- a short term extrapolation, of the duration of two or three satellite orbits, obtained beginning with the last known position of the balloon by the knowledge of the derivatives $\dot{\varphi}$ and $\dot{\lambda}$ of this position.

- a longer term extrapolation on the order of 24 H. That is possible if one utilizes a wind chart obtained from results of the scientific examination of the experiment.

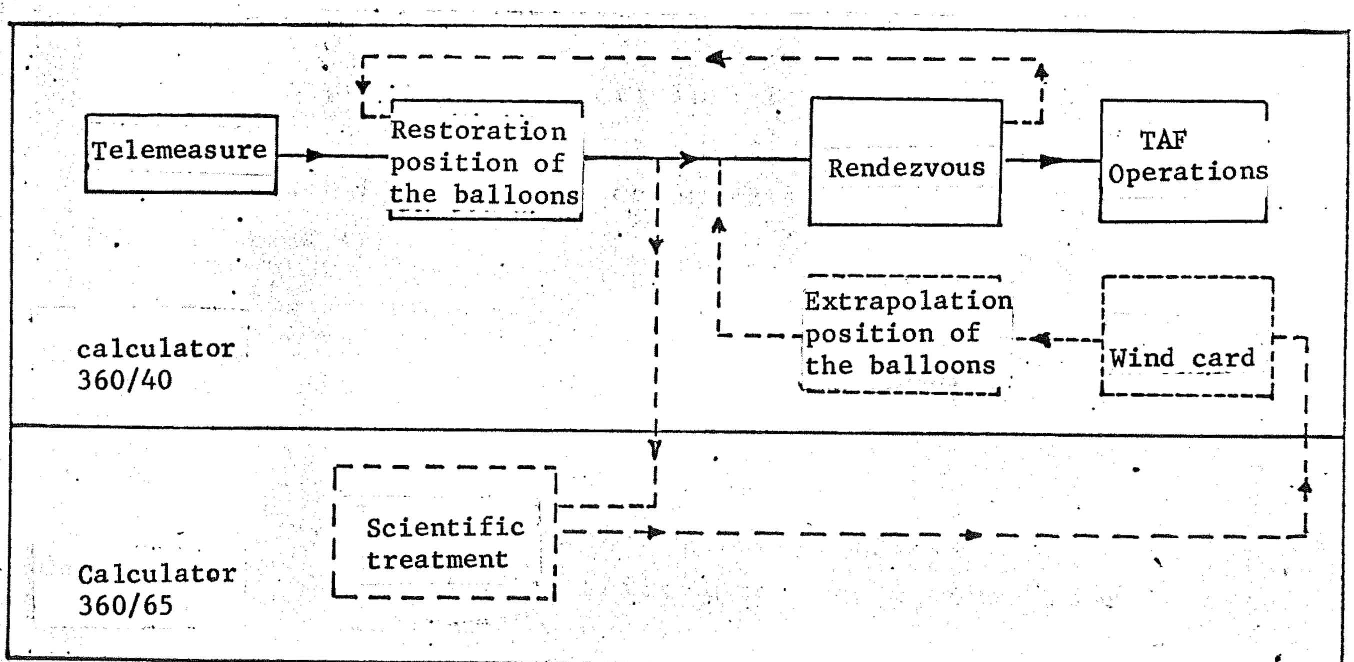


FIG. 1

The above schema indicates the organization of the treatment: the solid line corresponds to the first hypothesis, the dotted line corresponds to the second hypothesis.

The first organization has the advantage of simplicity. It only depends on the telemeasure, it only works out the treatment on the 360/40 calculator without depending on calculations worked out on another machine. It has the inconvenience of leading to imprecision on the calculation of the rendezvous for the balloons which would not have been able to be interrogated recently and does not permit establishment of a programmed plan of operation beyond two or three orbits. This organization of the treatment is actually at the stage of integration in the chain of treatment.

The second organization, which extrapolates the balloons' positions beginning with a wind chart, has the advantage of leading to a more precise plan of operation permitting the programmed interrogation of the balloons for more extensive periods. The knowledge of the extrapolated positions of the balloons at the moment of rendezvous possesses a double interest: on the one hand it furnishes initial solutions of good quality to the program of restoration of the positions which should permit reduction of working time; on the other hand, the comparison of the extrapolated positions with the restored positions beginning with measures test the validity of the scientific treatment. It possesses, however, non-negligible inconveniences:

- it depends on a periodic treatment carried out every 24 H on another machine;
- it weighs itself down with a numerical integration program for the extrapolation of the balloon positions which can be difficult to work on the 360/40 calculator.

This organization is, for the moment, only a project, the program of balloon position extrapolation being the object of conversations between the Mathematics and Treatment Division of the CENS and the team of Professor Morel in charge of scientific examination.

II Formalization of the balloon-satellite rendezvous

II-1 The existing preview programs are poorly adapted to take place in the chain of EOLE treatment. It is necessary to rethink the formalization of them in order to realize the following constraints:

- the encumbrance in memory calculator should be reduced,
- the execution times ought to be as short as possible realizing the great number of balloons.

For that, it is possible to elaborate a formalization which utilizes the simplifications brought by the quasi circularity of the orbit.

II-2 For the moment, we shall suppose that balloon B of geographical coordinates ϕ, Λ, H is immobile on the sphere of balloons, we shall next examine how to realize its new movement.

In an inertial mark, the vector coordinates $\vec{OB}$ are:

$$(1) \quad \vec{OB} \begin{cases} X_o = \rho \cos \phi \cos (\theta - \Lambda) \\ Y_o = \rho \cos \phi \sin (\theta - \Lambda) \\ Z_o = \rho \sin \phi \end{cases} \qquad \begin{array}{l} \theta = \text{sidereal Greenwich time} \\ \rho = R_T(\phi) + H \end{array}$$

Let's consider the orthonormal mark connected to the orbit plan containing the normal to the plan and the line of the nodes:

$$\overrightarrow{OB} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{pmatrix} \circ \begin{pmatrix} \cos\Omega & \sin\Omega & 0 \\ -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \circ \begin{pmatrix} X_o \\ Y_o \\ Z_o \end{pmatrix}$$

$$(2) \quad \overrightarrow{OB} = \begin{cases} X = \rho \cos u \cos\phi \\ Y = \rho \sin u \cos\phi \cos i + \rho \sin\phi \sin i \\ Z = -\rho \sin u \cos\phi \sin i + \rho \sin\phi \cos i \end{cases} \qquad \begin{array}{l} u = \theta - \Lambda - \Omega \\ \rho = \|\overrightarrow{OB}\| \end{array}$$

$$(2') \quad \overrightarrow{OB} = \begin{cases} X = P \cos u & P = \rho \cos\phi & \\ Y = Q \sin u + S & Q = \rho \cos\phi \cos i & S = \rho \sin\phi \sin i \\ S = R \sin u + T & R = \rho \cos\phi \sin i & T = \rho \sin\phi \cos i \end{cases}$$

According to the immobility hypothesis of the balloon, P,Q,R,S,T are constants, the time only intervenes in the variation of the angle u principally by the variation of θ sidereal time of the original meridian and secondarily by the variation of Ω.

$$\theta(t) = \theta_o + K(t-t_o) \qquad K = \frac{366.2422}{365.2422}$$

$$\Omega(t) = \Omega_o + \Omega_1(t-t_o)$$

$$(3) \quad u(t) = (\theta_o - \Omega_o - \Lambda) + (K - \Omega_1)(t-t_o)$$

The place of point H, projection of B on the orbital plan is then in ellipse of parametric equations:

$$(4) \qquad OH \begin{cases} X = P \cos u \\ Y = Q \sin u + S \end{cases} \qquad \text{(see figure 2)}$$

II - 3 Let's consider the plane perpendicular to the extremity of the vector $\vec{OB}$ which we will name horizon plane of the balloon B; its vectorial equation:

$$\vec{OB} \cdot \vec{BM} = 0$$

that is:

$$(x-X)X + (y-Y)Y + (z-Z)Z = 0$$

$$X\,x + Y\,y + Z\,z - \rho^2 = 0$$

The balloon will be seen by the satellite when this latter will be above the horizon plane of the balloon. In general, the horizon plane and the orbital plane are cut according to a straight line (D) which we shall call straight horizon, whose equation is:

$$Xx + Yy - \rho^2 = 0 \qquad Z = 0$$

Thus: $\cos\alpha\, x + \sin\alpha\, y - d = 0$ normal equation of (D)

With:

$$\alpha = \text{Angle } (X, Y) \qquad \text{(see figure 2)}$$

$$(5) \qquad d = \frac{\rho^2}{\sqrt{X^2 + Y^2}}$$

If p is the foot of the lowered normal of the origin on (D)

$$\vec{OP} \begin{cases} x_p = d \cos\alpha \\ y_p = d \sin\alpha \end{cases}$$

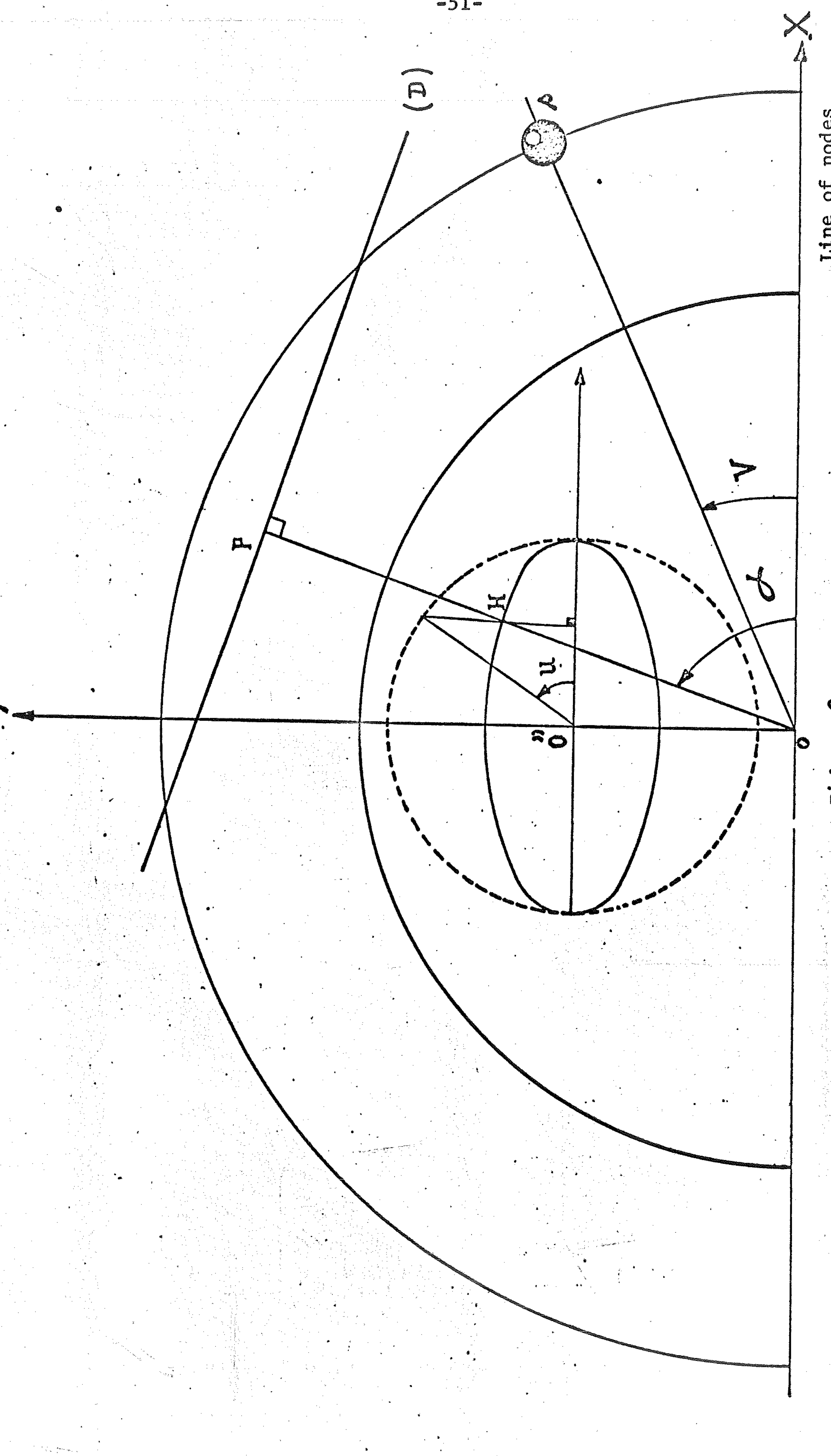


Figure 2

The vectors $\overrightarrow{OH}$ and $\overrightarrow{OP}$ are colinear and such that:

(5') $\qquad \overline{OP} \cdot \overline{OH} = \rho^2$

The location of p is obtained then as the transformation from the location of H by a power inversion p^2.

II - 4 To strictly determine the moments of appearance and disappearance of the satellite in view of the balloon, it would be necessary to calculate the moments where the satellite coincides with the points of orbital intersection and the straight horizon. This method can be simplified in the case of a circular orbit.

Let's consider the vectors:

$$\overrightarrow{OH} \begin{cases} X = P \cos u \\ Y = Q \sin u + S \end{cases} \qquad \overrightarrow{OS} \begin{cases} x = a \cos V \\ y = a \sin V \end{cases}$$

a = half large axis

$V = \omega + v = (n + \omega_1)\ (t - t_o)$ circular orbit

The satellite will certainly be visible if the following three geometric conditions are accomplished:

$$(6) \begin{cases} 1)\ \overrightarrow{OS} \wedge \overrightarrow{OH} = 0 \\ 2)\ \overrightarrow{OS} \cdot \overrightarrow{OH} > 0 \\ 3)\ \|\overrightarrow{OH}\| > \dfrac{p^2}{a} \Longleftrightarrow \|\overrightarrow{OP}\| < a \quad \text{relation (5')} \end{cases}$$

The relations (6-1) and (6-2) express the vector coincidence turning $\overrightarrow{OS}$ and $\overrightarrow{OH}$, the relation (6-3) expresses the intersection condition of the straight horizon with the circular orbit. Let's develop (6-1):

(6'-1): $(P \cos u) \sin V - (Q \sin u + S) \cos V = 0$

In this equation u and V are functions of the time, but in the case of the EOLE satellite, the angle V varies about 15 times more quickly than the angle u, this leads to consideration of solving the equation (6) by a method of successive approximations.

$$(7)\quad \begin{cases} \Delta t_i = \dfrac{(V_i - V_o)}{(n + \omega_1)} \\ u_i = u_o + (K - \Omega_1)\ \Delta t_i \\ V_{i+1} = \operatorname{arctg}\left(\dfrac{Q \sin u_i + S}{P \cos u_i} \right) \end{cases}$$

Let's eliminate the time, it comes:

$$(7')\quad \begin{cases} u_i = \left[u_o - V_o \left(\dfrac{K - \Omega_1}{n + \omega_1} \right) \right] + \dfrac{K - \Omega_1}{n + \omega_1}\ V_i = A + B\ V_i \\ V_{i+1} = \operatorname{arctg} \left[\dfrac{Q \sin (A + B\ V_i) + S}{P \cos (A + B\ V_i)} \right] \end{cases}$$

It concerns now demonstrating that the repetitive process converges:

that is to say that the method of the successive approximations solves the equation:

$$V = \operatorname{arctg}\left[\frac{Q \sin (A+B\,V) + S}{P \cos (A+B\,V)}\right] = \operatorname{arctg}\left(\frac{Y}{X}\right)$$

The equation is of the form $V = f(v)$ with $V \in [0,\ 2\pi]$: in order to establish the existence and uniqueness of the solution, we are going to show, since it is standard to do so, that the application $f(v)$ is contracting on a compact interval. It is necessary for that to find a uniform increase of the derivative:

$$f'(V) = \frac{XY' - X'Y}{X^2 + Y^2} = B\,\frac{P\left[Q + S \sin (A+BV)\right]}{\|\overrightarrow{OH}\|^2}$$

To increase $f'(V)$, decrease the denominator and increase the numerator. In effect, according to the relation (6-3), the denominator:

$$X^2 + Y^2 = \|\overrightarrow{OH}\|^2 > \frac{\rho^4}{a^2}$$

geometrically, that returns to exclude the point of ellipse location of H which are interior to the radius circle ρ^2/a which isolate the singular point of origin of $f'(V)$. As for the numerator:

$$BP\left[Q + S \sin (A+BV)\right] \leqslant B\,|P|\,(\,|Q| + |S|\,) \leqslant B\,\rho^2$$

whence =

$$|f'(V)| \leqslant B\,\frac{a^2}{\rho^2}$$

But

$$B = \frac{K - \Omega_1}{n + \omega_1} \simeq \frac{K}{n} \simeq \frac{2\pi}{86400}\sqrt{\frac{a^3}{\mu}}$$

for:

$$\begin{cases} n^2 a^3 = \mu \text{ (Kepler's 3rd law)} \\ \mu = 0.398607.10^6 \text{ km}^3 / \text{s}^2 \\ 86400 = \text{number of seconds in an average solar day.} \end{cases}$$

thus:

$$\left| f'(V) \right| \leq \frac{2\pi}{86400 \sqrt{\mu}} \qquad \frac{a^{7/2}}{\rho^2}$$

if we take:

$$\rho = 6378 \text{ km}$$

$$a = 7378 \text{ km} \Longleftrightarrow H_S = 1000 \text{ km}$$

$$\left| f'(V) \right| \leq 0.097$$

We have then a uniform increase of f'(V), which assures us of the existence and the uniqueness of the solution.

II - 4 We are now going to show with what approximation the moment where the conditions (6) are realized can be confused with the moment where the satellite culminates above the balloon.

The satellite culminates when the height "h" is maximum, or, that which is equivalent, the zenithal distance Δ is minimum ($h + \Delta = \frac{\pi}{2}$)

but:

$$\cos \Delta = \frac{\vec{OB} \cdot \vec{BS}}{\| \vec{OB} \| \, \| \vec{BS} \|}$$

with:

$$\begin{cases} \vec{BS} = \vec{OS} - \vec{OB} \\ \| \vec{OB} \| = \rho \text{ (constant)} \qquad \| \vec{OS} \| = a \text{ (circular orbit)} \end{cases}$$

whence:

$$\frac{d\Delta}{dt} = \left(\frac{\| \vec{BS} \|}{\| \vec{OB} \wedge \vec{OS} \|} \right) \frac{d}{dt} \left(\frac{\vec{OB} \cdot \vec{BS}}{\| \vec{BS} \|} \right)$$

while utilizing Lagrange identity, besides:

$$\begin{cases} \dfrac{d}{dt}(\vec{OB}).\vec{OB} = 0 \\ \\ \dfrac{d}{dt}(\vec{OS}).\vec{OS} = 0 \\ \\ \dfrac{d}{dt}(\vec{BS}).\vec{BS} = \dfrac{d}{dt}\|\vec{BS}\|\,\|\vec{BS}\| \end{cases}$$

It comes: (8) : $\dfrac{d\Delta}{dt} = \left(\dfrac{\vec{OS}\,.\,\vec{BS}}{\|OB \wedge OS\| \quad \|BS\|^2}\right)\dfrac{d}{dt}(\vec{OB}\,.\,\vec{OS})$

It is the vectoral equation of culmination. It is necessary to make some remarks:

1) $\|\vec{BS}\| \neq 0$ the satellite does not meet the balloon!

2) $\vec{OS}.\vec{BS} \neq 0$ according to the location of point H, projection of B on the orbital plan would intersect the orbit, but: $\|\vec{OH}\| < \rho <$ a

3) $\vec{OB} \wedge \vec{OS}=0 \Longrightarrow \vec{OS} = \lambda \vec{OB}$ whence:

$$\frac{d}{dt}(\vec{OB}.\vec{OS}) = \lambda \vec{OB}.\frac{d}{dt}(\vec{OB})+\frac{1}{\lambda}\vec{OS}.\frac{d}{dt}(\vec{OS}) = 0$$

the second member of (8) takes the indeterminant form $\frac{0}{0}$,

however the satellite culminates since it passes to the zenith, thus, in every case, we shall write the condition of culmination:

$$(9) \qquad \frac{d}{dt}\left(\vec{OB} \cdot \vec{OS}\right) = 0$$

Let's develop the relation (9) in the trihedron connected to the orbit previously utilized which we shall consider as fixed:

$$\frac{d}{dt}(\vec{OB}) \cdot \vec{OS} + \frac{d}{dt}(\vec{OS}) \cdot \vec{OB} = 0$$

$$\frac{d(\vec{OB})}{dt} \begin{cases} X' = -\dot{u}\, P \sin u \\ Y' = \dot{u}\, Q \cos u \\ Z' = \dot{u}\, R \cos u \end{cases} \qquad \frac{d(\vec{OS})}{dt} \begin{cases} x' = -\dot{V}\, a \sin V \\ y' = \dot{V}\, a \cos U \\ z' = 0 \end{cases} \qquad B = \frac{\dot{u}}{\dot{V}}$$

$$(9') \qquad B\left[(-P \sin u) \cos V + (Q \cos u) \sin V\right] + \left[(Q \sin u + S) \cos V - (P \cos u) \sin V\right] = 0$$

In this relation the quantity included in the second turn is identical to the first number of the relation (6'-1) which expressed the colinearity of the vectors $\vec{OH}$ and $\vec{OS}$:

$$(6'\text{-}1) \qquad X \cos V - Y \sin V = 0$$

Let's put

$$\vec{OH} \begin{cases} X = P \cos u \\ Y = Q \sin u + S \end{cases} \qquad \vec{HH'} \begin{cases} \Delta X = -\, BP \sin U \\ \Delta Y = -\, BQ \cos U \end{cases}$$

We can write (9') in the form:

$$(9') \qquad (X + \Delta X) \cos V - (Y + \Delta Y) \sin V = 0$$

Let's evaluate the error ΔV committed on V = angle (X,Y) while considering ΔX and ΔY as errors on X and Y.

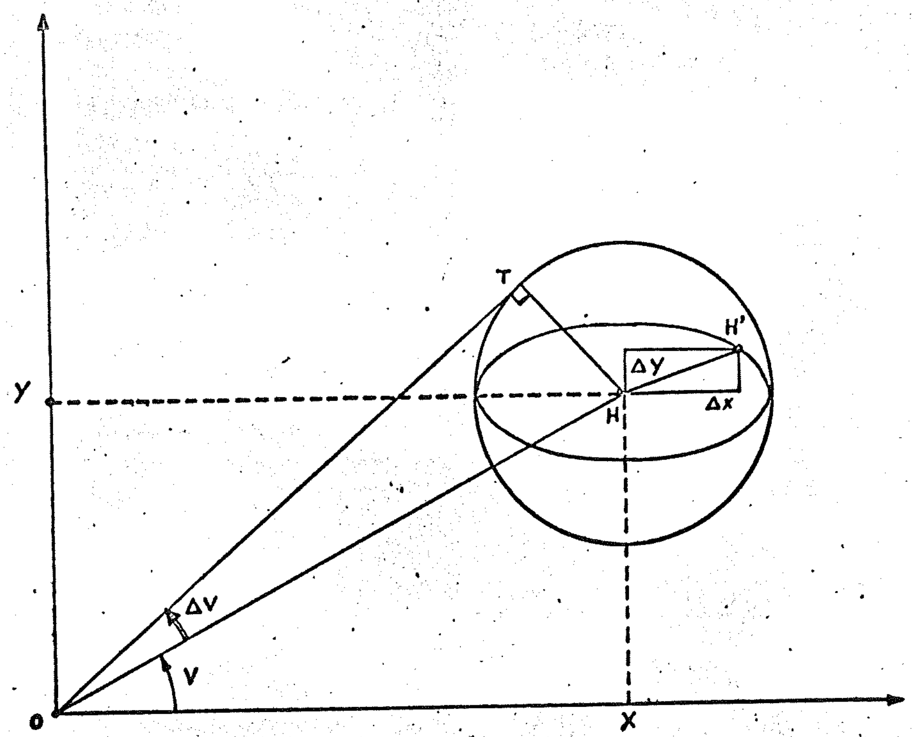


Fig.3

Let's notice that the vector of the components ΔX and ΔY describes an ellipse, of half large axis BP and of half small axis BQ, whence

$$\|\overrightarrow{HH'}\| = (\Delta X^2 + \Delta Y^2)^{1/2} = B(P^2 \sin^2 u + Q^2 \cos^2 u)^{1/2} < BP = B\rho\cos\phi$$

The error committed on V is the angle under which it is seen from O the vector $\overrightarrow{HH'}$, this error is increased by the half angle under which it is seen from O the circle of center H and of radius $\rho \cos \phi$ (see figure 3).

We can then write:

$$\sin \Delta V \leqslant \frac{\|\overrightarrow{TH}\|}{\|\overrightarrow{OH}\|} = \frac{B \rho \quad \cos \Phi}{(X^2+Y^2)^{1/2}}$$

But, we must verify, in order that there be a visible passage:

$$\|\overrightarrow{OH}\| = (X^2+Y^2)^{1/2} \geqslant \frac{\rho^2}{a}$$

Whence:

$$\sin \Delta V \leqslant \frac{aB \cos \Phi}{\rho} \leqslant \frac{aB}{\rho}$$

As previously:

$$B = \frac{2 \pi}{86400} \sqrt{\frac{a^3}{\rho}}$$

Whence:

$$\sin \Delta V = \frac{2 \pi}{86400 \sqrt{\mu}} \quad \frac{a^{5/2}}{\rho}$$

If, as previously, we take:

$$\begin{cases} p = 6378 \text{ km} \\ a = 7378 \text{ km} \end{cases}$$

$$\sin \Delta V \leqslant 0.0844$$

$$\Delta V \leqslant 4.85^\circ \Longleftrightarrow \Delta t \leqslant 85 \text{ seconds}$$

II - 5 Let V be the angle which verifies the relations (6), the corresponding straight horizon is known by its normal coordinates, that is to say by the polar coordinates of the vector $\overrightarrow{OP}$, or thus:

$$\beta = \text{Arcos } \left(\frac{\|\overrightarrow{OP}\|}{a} \right)$$

We shall define the angles corresponding to the appearance and the disappearance of the satellite by:

$$\begin{cases} V_L = V - \beta \\ V_C = V + \beta \end{cases}$$

This is not strictly exact in the study made in paragraph II-4 indicates to us the order of greatness of the error committed.

However, this is sufficient to determine the intervals of time where the satellite is visible, the rapidity of the process of calculation compensating its relative imprecisions.

In the first version of the preview program, we shall content ourselves with this method. The latter can be improved later by looking for solutions to the equation (9') which can be substituted in the equation (6'-1), the analogy of form of these relations not setting back in cause the principle of the method.

II - 6 The preceding calculations are accomplished beginning from a position Λ, Φ of the balloon. This latter, in accordance with what has been said in Chapter I, can be:

- either the last restored position
- or the extrapolated position as far as the nearest passage to the descending node.

The index of the rendezvous will be composed of the union of the corresponding subindexes giving for each balloon the rendezvous orbit by orbit. Each one of these subindexes being made up of information classified in the chronological order which will permit, by methods of direct access, establishment of a plan of operation for each orbit.

The information inscribed on the index, for each orbit, will be:

NUB, NOR, IND, DAT1, DAT2, HZM, DT, FI, AL, FI1, AL1

NUB:	number of the balloon
NOR:	number of the orbit
IND:	index = 1 if the position of the balloon has been extrapolated 0 if the position of the balloon is the last restored -1 if on the corresponding orbit there is no passage.
DAT1:	date of the passage at maximum height in entire number of julien days
DAT2:	fraction of julien day corresponding to the passage at maximum height
HZM:	maximum height
DT:	interval of time in a julien day, between the passage at maximum height and the rising or setting.
FI:	latitude of the balloon, in radian
AL:	longitude of the balloon in radian
FI1:	derives from the latitude in radian per day
AL1:	derives from the longitude in radian per day